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NEWTON–COTES FORMULAS FOR NUMERICAL INTEGRATION IN MAPLE

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Abstract. This article discusses the Newton-Coates formulas for the numerical integration of the Maple package, which have been discussed. In this study, for the purpose of numerical integration of trapezoidal method, Simpson method and Simpson 3/8 method, we examined. We analyzed the function f defined in the interval [a, b] at point f (x) by presenting several analyzes. The Maple package provides the latest applications of numerical analysis in a shorter and more efficient way, which has led to the solution of important scientific and technical problems today. With the help of Maple software, we solved our desired changes on its clear and reliable forms.

Key words: Maple, Newton-Coates formula, Trapezoid, Simpson's Rule, Simpson's 3/8 Rule.

Introduction

Computing software has advanced a lot in recent decades. Today, computing software is used not only in specialized work, but also in educational affairs, academic research and textbook writing. Founded in the early 1980s at the University of Waterloo, Canada, Maple software, in newer versions, offers many computational problems with accurate answers. This software is easy to learn because the same math symbols used in classrooms can be used to enter data. Numerical analysis is one of the most important methods that is very important for performance in many algorithms. Modern numerical analysis tries to understand the data in a shorter or more concise way. Any approximate method must converge to the correct answer [7].

Today, using the latest applications of numerical analysis, they try to calculate the data in a shorter and more efficient way. The Maple package provides numerical methods that lead to the solution of important scientific and technical problems. Maple package also has features for input and output of image and output files. We analyzed the Newton-Coates formula in the Maple package considering other methods. Integration is the technique of calculating the area plotted on a graph using a function [1].

$$I = \int_a^b f(x)\,dx$$

Materials and methods

Because of these tasks, interpolation-based numerical integration using the following methods is accurately demonstrated by Maple.

A. NEWTON–COTES FORMULA

We study Newton-Coates formulas using some Maple computational capabilities with respect to the approximate function at distances [a, b] for the integral f (x) [2].

Examples

> $\int_{1.1}^{8.0} \ln(x)\,dx$

9.630691136

> $with(Student[Calculus1])$:

> $ApproximateInt(\ln(x), x = 1.1 ..8.0, method = newtoncotes[1]);$

9.600011676

> $ApproximateInt(\ln(x), x = 1.1 ..8.0, method = newtoncotes[2]);$

9.630586303

> $ApproximateInt(\ln(x), x = 1.1 ..8.0, method = newtoncotes[3]);$

$$9.630643941$$

> $ApproximateInt(\ln(x), x = 1.1..8.0, method = newtoncotes[4]);$

$$9.630690519$$

> $ApproximateInt(\ln(x), x = 1.1..8.0, method = newtoncotes[6]);$

$$9.630691129$$

B. TRAPEZOIDAL METHODS

The trapezoidal rule is a Newton-Cotes formula for approximating the integral of a function f using linear segments. Let f be tabulated at points x_0 and x_1 spaced by a distance h, and write $f_n = f(x_n)$. then the trapezoidal rule states that $\int_{x_0}^{x_1} f(x)\,dx \approx h(f_0 + f_1)/2$.

Examples

> $polynomial := CurveFitting[PolynomialInterpolation]([x_0, x_1], [f(0), f(1)], z):$

> $integrated := \int_{x_0}^{x_1} polynomial\, dz:$

> $factor(integrated)$

$$-\frac{1}{2}(x_0 - x_1)(f(1) + f(0))$$

> $with(Student[Calculus1]):$

> $ApproximateInt(\sin(x), x = 3..5, method = trapezoid)$

$$\frac{1}{10}\sin(3) + \frac{1}{5}\sin\left(\frac{16}{5}\right) + \frac{1}{5}\sin\left(\frac{17}{5}\right) + \frac{1}{5}\sin\left(\frac{18}{5}\right) + \frac{1}{5}\sin\left(\frac{19}{5}\right) + \frac{1}{5}\sin(4) + \frac{1}{5}\sin\left(\frac{21}{5}\right) + \frac{1}{5}\sin\left(\frac{22}{5}\right) + \frac{1}{5}\sin\left(\frac{23}{5}\right) + \frac{1}{5}\sin\left(\frac{24}{5}\right) + \frac{1}{10}\sin(5)$$

> $ApproximateInt(\cos(x), 1..100, method = trapezoid, output = animation)$

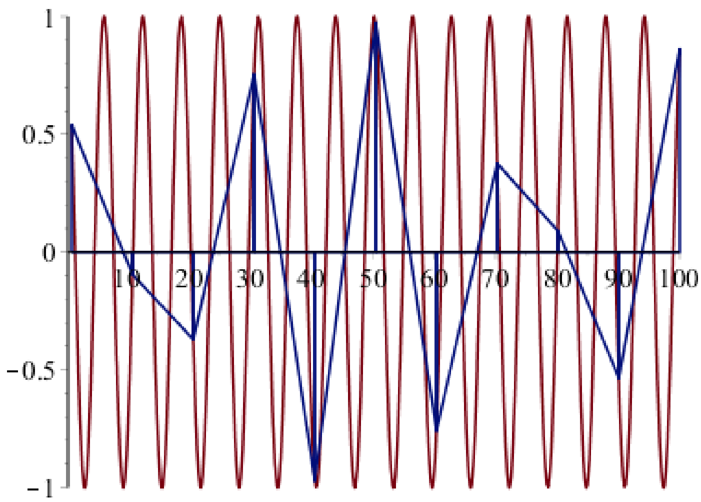

Figure 1 An animated approximation of $\int_{1}^{100} f(x)\,dx$ using trapezoid rule, where $f(x) = \cos(x)$ and the partition is uniform.
The approximate value of the integral is 1.615815314.
Number of subintervals used: 10.

C. SIMPSON'S RULE

Simpson's Rule is a numerically accurate method of approximating a definite integral usi
ng

a three - point quadrature obtained by integrating the unique quadratic that passes through these points [3,5]. for some c in the interval $[x_0, x_2]$, provided that $f\ (iv)$ exists and is continuous. Concluding the derivation yields Simpson's Rule [2,6]:

$$\int_{x_0}^{x_2} f(x)dx = \frac{h}{2}(y_0 + 4y_1 + y_2) - \frac{h^5}{90} f^{(iv)}(c).$$

Where $h = x_2 - x_1 = x_1 - x_0$ and c between x_0 and x_2.

Examples

> $polynomial := CurveFitting[PolynomialInterpolation]\left(\left[x_0, \frac{x_0 + x_1}{2}, x_1\right], \left[f(0), f\left(\frac{1}{2}\right), f(1)\right], z\right)$

> $integrated := \int_{x_0}^{x_1} polynomial\, dz :$

> $factor(integrated)$

$$-\frac{1}{6}(x_0 - x_1)\left(f(1) + 4f\left(\frac{1}{2}\right) + f(0)\right)$$

> $with(Student[Calculus1]) :$

> $ApproximateInt(\sin(x), x = 2..-3, method = simpson)$

$$-\frac{1}{6}\sin\left(\frac{5}{2}\right) - \frac{1}{3}\sin\left(\frac{11}{4}\right) - \frac{1}{12}\sin(3) - \frac{1}{12}\sin(2) - \frac{1}{3}\sin\left(\frac{9}{4}\right)$$

> $ApproximateInt(\cos(x), 1..100, method = simpson, output = animation)$

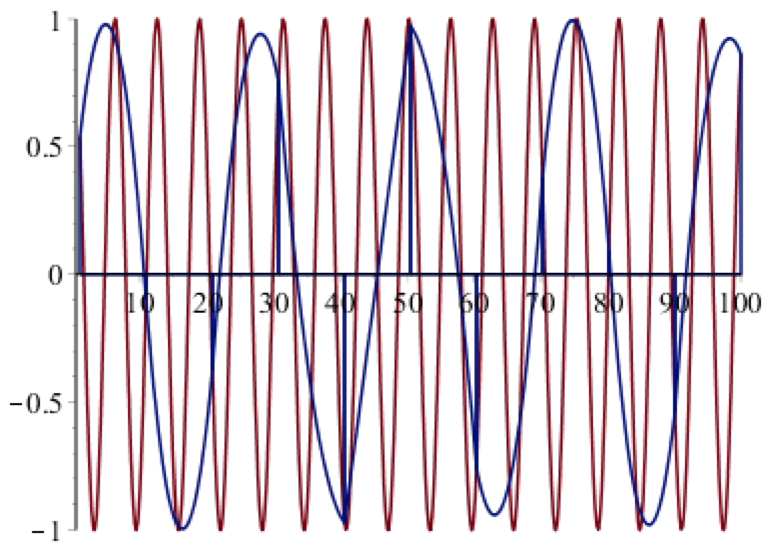

Figure 2 An animated approximation of $\int_1^{100} f(x)\, dx$ using Simpson's rule, where $f(x) = \cos(x)$ and the partition is uniform. The approximate value of the integral is 5.115050025. Number of subintervals used: 10.

D. THE GENERAL FORMULA OF SIMPSON'S 3/8 RULE

Let the values of a function $f(x)$ be tabulated at points x_i equally spaced by $h = x_{i+1} - x_i$, so $f_1 = f(x_1)$, $f_2 = f(x_2)$, ..., $f_4 = f(x_4)$. Then Simpson's 3/8 rule approximating the integral of $f(x)$ is given by the Newton-Cotes-like formula [7].

$$\int_{x_1}^{x_4} f(x)dx = \frac{3}{8}h(f_1 + 3f_3 + f_4) - \frac{3}{8}h^5(\xi).$$

This method is completely based on cubic interpolation [8]. With this method, we demonstrated some computational capabilities of the Maple package [4].

Examples

$$> polynomial := CurveFitting[PolynomialInterpolation]\left(\left[x_0, \frac{2x_0+x_1}{3}, \frac{x_0+2x_1}{3}, x_1\right], \left[f(0), f\left(\frac{1}{3}\right), f\left(\frac{2}{3}\right), f(1)\right], z\right):$$

$$> integrated := \int_{x_0}^{x_1} polynomial\, dz :$$

$$> factor(integrated)$$

$$-\frac{1}{8}(x_0 - x_1)\left(f(1) + 3f\left(\frac{2}{3}\right) + 3f\left(\frac{1}{3}\right) + f(0)\right)$$

$$> with(Student[Calculus1]) :$$

$$> ApproximateInt\left(\sin(x), x = 1..6, method = simpson_{\frac{3}{8}}\right)$$

$$\frac{1}{8}\sin\left(\frac{11}{2}\right) + \frac{3}{16}\sin\left(\frac{17}{3}\right) + \frac{3}{16}\sin\left(\frac{35}{6}\right) + \frac{1}{16}\sin(6) + \frac{3}{16}\sin\left(\frac{31}{6}\right) + \frac{3}{16}\sin\left(\frac{16}{3}\right) + \frac{1}{8}\sin(5) + \frac{3}{16}\sin\left(\frac{23}{6}\right) + \frac{1}{8}\sin(4) + \frac{3}{16}\sin\left(\frac{25}{6}\right) + \frac{3}{16}\sin\left(\frac{13}{3}\right) + \frac{1}{8}\sin\left(\frac{9}{2}\right) + \frac{3}{16}\sin\left(\frac{14}{3}\right) + \frac{3}{16}\sin\left(\frac{29}{6}\right) + \frac{3}{16}\sin\left(\frac{17}{6}\right) + \frac{1}{8}\sin(3) + \frac{3}{16}\sin\left(\frac{19}{6}\right) + \frac{3}{16}\sin\left(\frac{10}{3}\right) + \frac{1}{8}\sin\left(\frac{7}{2}\right) + \frac{3}{16}\sin\left(\frac{11}{3}\right) + \frac{3}{16}\sin\left(\frac{5}{3}\right) + \frac{3}{16}\sin\left(\frac{11}{6}\right) + \frac{1}{8}\sin(2) + \frac{3}{16}\sin\left(\frac{13}{6}\right) + \frac{3}{16}\sin\left(\frac{7}{3}\right) + \frac{1}{8}\sin\left(\frac{5}{2}\right) + \frac{3}{16}\sin\left(\frac{8}{3}\right) + \frac{3}{16}\sin\left(\frac{7}{6}\right) + \frac{3}{16}\sin\left(\frac{4}{3}\right) + \frac{1}{8}\sin\left(\frac{3}{2}\right) + \frac{1}{16}\sin(1)$$

$$> ApproximateInt\left(\cos(x), 1..100, method = simpson_{\frac{3}{8}}, output = animation\right)$$

The results show that the Maple package has good computational capabilities and can be very useful for analyzing generalized numerical integration methods of a software. We tested some of the computational capabilities of the Maple package using the Newton-Coates formula for numerical integration.

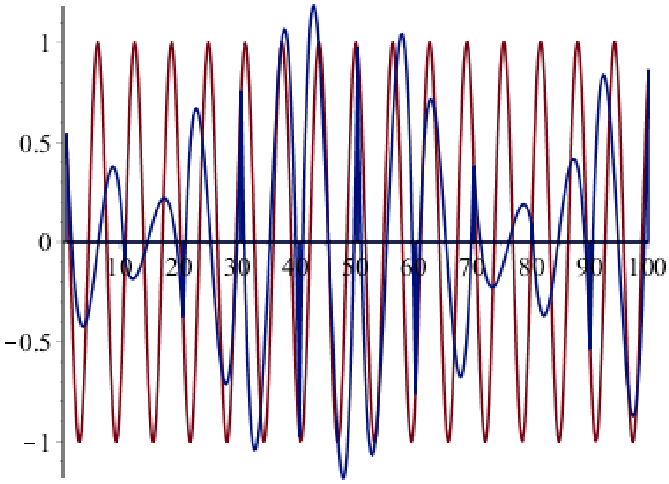

Figure 3 An animated approximation of $\int_{1}^{100} f(x)\,dx$ using Simpson's 3/8 rule, where $f(x) = \cos(x)$ and the partition is uniform. The approximate value of the integral is −0.003400110048. Number of subintervals used: 10.

Conclusion

The results show that the Maple package has good computational capabilities and can be very useful for analyzing generalized numerical integration methods of a software. We tested some of the computational capabilities of the Maple package using the Newton-Coates formula for numerical integration.

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MAPLE-ГЕ САНДЫҚ ИНТЕГРАЦИЯҒА АРНАЛҒАН НЬЮТОН-КОТС ФОРМУЛАЛАРЫ

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Аңдатпа. Бұл мақалада қазірдің өзінде талқыланған Maple пакетін сандық интеграциялауға арналған Ньютон-Коутс формулалары қарастырылады. Бұл зерттеуде біз сандық интеграция мақсатында трапеция әдісін, Симпсон әдісін және Симпсон 3/8 әдісін қарастырдық. Біз f (x) нүктесінде [A, b] интервалында анықталған f функциясын талдадық, бірнеше талдауларды ұсындық. Бүгінгі таңда Maple пакеті сандық талдаудың соңғы қосымшаларын қысқа және тиімді түрде ұсынады, бұл маңызды ғылыми және техникалық мәселелерді шешуге әкелді. Maple бағдарламалық жасақтамасының көмегімен біз оның түсінікті және сенімді формаларына қажетті өзгерістер енгіздік.

Түйін сөздер: Үйеңкі, Ньютон-Коутс формуласы, Трапеция, Симпсон ережесі, Симпсон ережесі 3/8.

ФОРМУЛЫ НЬЮТОНА–КОТСА ДЛЯ ЧИСЛЕННОГО ИНТЕГРИРОВАНИЯ В MAPLE

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Абстракт. В этой статье обсуждаются формулы Ньютона-Коутса для численного интегрирования пакета Maple, которые уже обсуждались. В этом исследовании мы рассмотрели трапециевидного метода, метода Симпсона и метода Симпсона 3/8 с целью численного интегрирования. Мы проанализировали функцию f, определенную в интервале [a, b] в точке f (x), представив несколько анализов. Сегодня пакет Maple предоставляет новейшие приложения численного анализа более коротким и эффективным способом, что привело к решению важных научных и технических проблем. С помощью программного обеспечения Maple мы внесли желаемые изменения в его понятные и надежные формы.

Ключевые слова: Клен, формула Ньютона-Коутса, Трапеция, Правило Симпсона, Правило Симпсона 3/8.

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