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MODERN REVIEW OF PAST PROBLEMS IN APPLIED MATHEMATICS AND COMPUTER SCIENCE

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Abstract. In this article we present the novel model of studying the past problems in present. These problems are very well handled by many authors; however, the result remains unproved. The problems are as follows: Nash equilibria in co-operative games and P versus NP theorem by Stephen Cock. We show that there is a solution for both classical problem in a partial case for "P versus NP"-theorem and co-operative games equilibria for all cases. Since partial case for P-NP problem could be proved by showing that Bellman's dynamic programming (DP) is the most optimal algorithm for composite tasks and problems. We also show that same equation by Bellman within the pre-defined parameter can be valid for both P and NP classes of problem according to the ordered sets of arbitrary variables which are compound to Bellman's equation, which was studied well in prior works by the same author, who holds the position of IT-analyst at the present time.
Keywords: Nash equilibria, computational complexity, P versus NP, proof of concept.

Introduction

For the past time the Nash equilibria and economical drastically known results for co-operative games were well studied and, even, honored as a Nobel Prize; in this article we present the final equation for the past work and show that the equilibria is commonly tied to the meaning of probability and statistics, which is handled well by Applied Mathematics as one of the most important scientific disciplines – this fact is well-known by author due to Prof. Vinoo Cameron, who presently resides and United State of America (USA) and holds Medicine Doctor (MD) position at the Hope Clinic, Athens.

Meanwhile many researchers see a valuable perspective of the result proposed by John Forbes Nash [1].

More results are to come for showing that Nash equilibria from the computational point of view is NP-complete and, thus, cannot be handled by modern era computing technologies – this fact is due to the recurrently dependent nature of the Nash algorithm for co-operative games with only two sides as minimal. We prove the result for many players of these economical market games, which is an extended case and, thus, cannot be omitted. This question was first put in the science in [2]. Many other attempts were made towards the open question of the Nash equilibria for the arbitrary size of set of players [3, 4]. From all the above, we define the probability of the Nash theorem for the common or "full" case – for this purpose we use the statistical probability for each player.

Since we have worked out our concerns about Nash equilibria, it's time for another important question in Computer Science. As to author's concerns this question is somehow tied to the mathematics in common and is included as one of the Millennium Problem by the Clay Institute of Mathematics, along with, for example, Navier-Stocks equation and other most important problems well-studied and known in the modern age [5]. This problem is originated from the official theory by the Alan Mathison Turing Award recipient Stephen Cook [6]. We begin our study of this naturally important question from previous publication [7]. We also give a notion to the modern works for "P versus NP" theorem by Lance Fortnow [8] and M. Sipser [9], which are the one of the latest known results to the present time: to the opinion of Dr. Gennady Fedulov from ResearchGate™, most of the researchers consider the P-NP question unsolvable due to the poor theoretical background for the definition of the algorithm and NP-complete problem, however, the author of the past work [7] gives the definition of theory and practice for the term “algorithm”. We will use the Bellman algorithm [10] as the starting point for the successful evaluation of the modern result, described in this work. We will also show further that according to Bellman results the algorithms can be successfully classified as P and NP complete.

Nash equilibria in Co-operative Games. The classical description of the co-operative game in economics and other financial sciences is due to the winning rate in case of getting the step on the matrix n x n for two players, where at each step the new row or column is taken by one of the players in periodical order: the values of winning or loosing are given as is by this matrix.

We state that the composition of the Nash equilibria or problem is NP-complete, since there are almost factorial number of combinations for the definition of the target function which by the opinion of John Forbes Nash gives the minimal difference between the score of the game for both players.

We also state that this opinion, besides the computational complexity of the “task”, was misinterpreted in past and for now the general case for n-players cannot be omitted.

We present the result according to probability model and the following consequence:

$$r_i = \frac{v_i}{\sum v_i} \cdot \sum s_i, \quad (1)$$

where R is a set of rational number, V is a set of values and S is a set of measures.

The equation defines the arbitrary function p(x) by the division operator of each value and sum of all values – this is a generalized case of the co-operative games, where the arbitrary number of players gain the positive result according to the pre-defined probability.

Thus, the main case holds true and isn’t NP-complete as the initial result obtained by Prof. John Forbes Nash.

The proof of the NP-completeness of the initial case as described above follows from the fact that, as it’s known, in the initial circumstances the function p(x) is given by the recurrent relation between each turn in a series recorded for both, or two, players – thus, the number of possible combinations to be considered leads to the computational explosion and is almost exponential and even, as stated before, is of, more preciously, factorial nature.

We propose the novel approach of solving the co-operative games for general case as the number of players is defined by the dimension of vector V in (1).

The proof of P-completeness of the general case and the similarity of the system of equations by Nash naturally follows from the re-formulation of problem and the definition of the statistical probability.

Thus, in this section we have shown the importance of the newly obtained results described in this work.

In the next section we will learn about the Millennium Problem proposed by Stephen Cook, which is remains open in the modern age.

P versus NP in Computational Complexity. We state that if there's an order in the target function for the set of variables, then, P doesn’t equal NP according to the dynamic programming which is the most optimal way of solving the combinatorial problems using its recurrent function at each step of the algorithm.

The problem was first stated by Stephen Cook, from that time on now we have no defined answer if there's the possibility of finding the polynomial solution for NP-complete problems.

We assume that the growing speed of polynomial and non-polynomial functions is compared to each other:

$$O(n^t) < O(2^n) < O(n!) < O(n^n), \quad (2).$$

From the equation (2) it also naturally follows the classification of P and NP complete classes like P-complete nt and exponential and further 2n.

Since there's no known algorithms whose polynomial speed is faster than the speed of growth of non-polynomial functions like power-set or factorial, then it naturally follows that if P and NP are equal, then speed of algorithm steps in P is equal or greater than number of steps required for the NP-complete task:

$$\frac{\partial x^t}{\partial x} \geq \frac{\partial 2^x}{\partial x} \rightarrow P = NP \quad , \qquad (3).$$

The same applies to the factorial function and its equivalent value as function in form xx. As t is a free parameter then we see that:

$$t \cdot x^{t-1} < \ln(2) 2^x \quad , \qquad (4).$$

The inequality (4) is true for factorial also.

Thus, from (3) and (4) it naturally follows that the P and NP classes are different as the speed of growing non-polynomial function is much bigger than the same speed defined by derivative of polynomial function in P.

The plot of P and NP functions along with their derivatives can be seen on Figure 1.

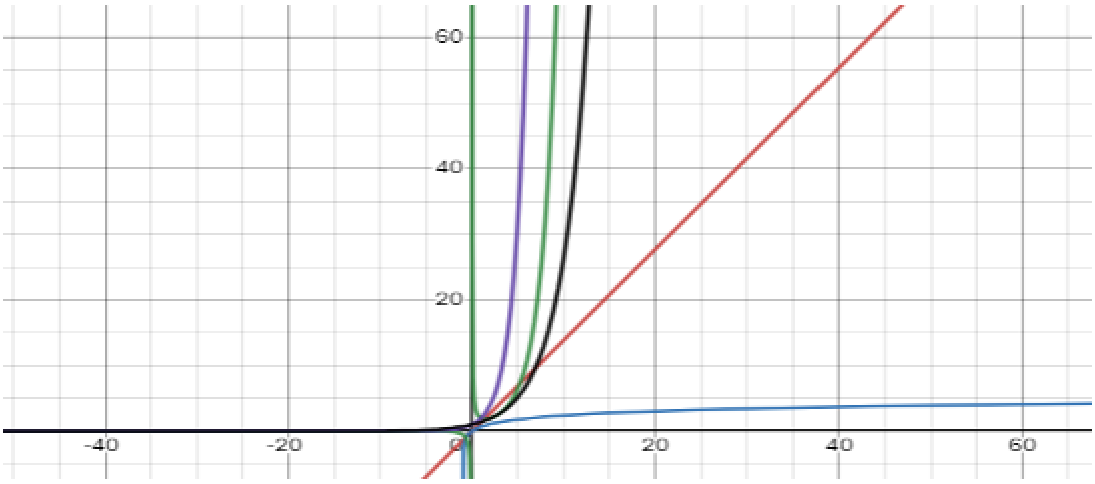

Figure 1 - Graphical plot of functions and derivatives

From all the above, it still doesn't follow that P != NP, however, for ordered set there's no optimal solution in P as the fastest possible algorithm's complexity is O(n) for iterative methodology, as it's presented at each step before inclusion at the stage set using dynamic programming, which is the most optimal as it's defined by recurrence relation at each stage.

We have shown that classes P and NP are different over the fact of derivatives defining the speed of the polynomial and non-polynomial functions respectively. It naturally follows that even for infinite order of derivative comparison the speed of non-polynomial functions is still increasing while the speed of polynomial function converges to absolute zero:

$$\lim_{n \to \infty} \frac{\partial^n x^t}{\partial t^n} < \lim_{n \to \infty} \frac{\partial^n 2^x}{\partial t^n} < \lim_{n \to \infty} \frac{\partial^n x^x}{\partial t^n} \quad , \qquad (5).$$

If there exist the ordered set in target function like in Traveling Salesman Problem (TSP), then P != NP according to the obvious statement that dynamic programming cannot cover all the space of possible cases.

The Bellman function is defined as follows for the set of variables without any order:

$$U_i(t, s_i) = arg_t\, opt \{ U_{i-1}(t - t_{0,} s_{i-1}) + F_i(t_0) \} \quad , \qquad (6)$$

Since, there's no order in the target function Un, the (6) gives the polynomial solution to the problem. However, when there's an order we cannot use the ordered set for optimization problem on tape position t on the Turing automaton and, thus, P != NP – this is the final proof of Millennium theorem.

Conclusion

In this work we have shown the general case of the Nash equilibria in co-operative games for arbitrary number of players and proposed the ways of proving the correctness of this statement basing upon the applied mathematics sciences like statistics and theory of probability.

We have also shown the way of considering the partial case for “P versus NP” theorem as per the dynamic programming algorithm by Bellman.

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ҚОЛДАНБАЛЫ МАТЕМАТИКА МЕН ИНФОРМАТИКАДАҒЫ ӨТКЕН МӘСЕЛЕЛЕРГЕ ЗАМАНАУИ ШОЛУ

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Аңдатпа. Бұл мақалада біз өткеннің қазіргі кездегі мәселелерін зерттеудің жаңа моделін ұсынамыз. Бұл мәселелерді көптеген авторлар өте жақсы шешеді; дегенмен, нәтиже дәлелденбеген болып қалады. Мәселелер келесідей: Нэштің кооператив ойындарындағы тепе-теңдігі және Стивен Коктың P-NP қатынасы туралы теоремасы. Біз "P vs NP" теоремасы үшін де, барлық жағдайларда кооператив ойындарының тепе-теңдігі үшін де классикалық есептің шешімі бар екенін көрсетеміз. P-NP тапсырмасының ішінара жағдайын Беллманның динамикалық бағдарламалауы (DP) күрделі есептер мен мәселелер үшін ең оңтайлы алгоритм екенін көрсету арқылы дәлелдеуге болады. Біз сондай-ақ алдын ала анықталған параметр шегінде бірдей Беллман теңдеуі АТ позициясын ұстанатын сол автордың алдыңғы жұмыстарында жақсы зерттелген Беллман теңдеуі үшін құрама болып табылатын реттелген ерікті айнымалылар жиынтығына сәйкес P және NP есептер кластары үшін жарамды болуы мүмкін екенін көрсетеміз-қазіргі уақытта талдаушы.

Кілттік сөздер: Нэш тепе-теңдігі, есептеу күрделілігі, NP-мен салыстырғанда P, тұжырымдаманың дәлелі.

СОВРЕМЕННЫЙ ОБЗОР ПРОШЛЫХ ПРОБЛЕМ В ПРИКЛАДНОЙ МАТЕМАТИКЕ И ИНФОРМАТИКЕ

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Аннотация. В этой статье мы представляем новую модель изучения проблем прошлого в настоящем. Эти проблемы очень хорошо решаются многими авторами; однако результат остается недоказанным. Проблемы заключаются в следующем: равновесие Нэша в кооперативных играх и теорема Стивена Кока о соотношении P и NP. Мы показываем, что существует решение как для классической задачи в частном случае для теоремы “P против NP”, так и для равновесий кооперативных игр для всех случаев. Поскольку частичный случай для задачи P-NP можно было бы доказать, показав, что динамическое программирование Беллмана (DP) является наиболее оптимальным алгоритмом для сложных задач и проблем. Мы также показываем, что одно и то же уравнение Беллмана в пределах заранее определенного параметра может быть справедливо как для P, так и для NP классов задач в соответствии с упорядоченными наборами произвольных переменных, которые являются составными для уравнения Беллмана, которое было хорошо изучено в предыдущих работах того же автора, который придерживается позиции ИТ-аналитик в настоящее время.

Ключевые слова: равновесия Нэша, вычислительная сложность, P в сравнении с NP, доказательство концепции.

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